Optimization of the "Coarse Grained Localization" Algorithm for Low Power in Wireless Sensor Networks

Jan Blumenthal, Frank Reichenbach, Matthias Handy, and Dirk Timmermann

Institute of Applied Microelectronics and Computer Science
University of Rostock
{jan.blumenthal;frank.reichenbach;matthias.handy;dirk.timmermann}
@etechnik.uni-rostock.de

Abstract. In this paper, we describe the optimization of techniques to determine the position of nodes in a wireless sensor network based on the "Coarse Grained Localization" algorithm. The algorithm features very low complexity at sufficient precision thus matching the low resources found in wireless sensor networks.
All nodes in a sensor network calculate their positions by center determination of received position information of few base stations with known positions. The main focus of our analysis is the determination of an optimal transmission range of all base stations for position determination prior installation of nodes. At an optimal transmission range, the highest precision is achieved whereas nodes require lowest energy.
We present analytic methods for adjusting the transmission range and configuring all parameters optimally. Thereby, we achieve a position determination with lowest communication and calculation effort given an allowed positioning error.

Keywords Coarse Grained Localization, Positioning, Center Determination, Wireless Sensor Network

1 Introduction

A sensor network consists of hundreds or thousands of tiny sensor nodes, which are randomly spread out over an area of interest. The objective is to measure values in the environment and propagate this information to data sinks of the system. I.F. Akyildiz gives a very comprehensive and detailed introduction on sensor networks in [1].
A sensor node typically consists of a battery, a microprocessor, a communication module, sensors, and/or actuators. Due to the desired node's size of some millimeters, the dimensions of the communication module and the battery are critical. Consequently, the scarcest resource within a network is the available energy. Therefore, it is essential to use low power optimized algorithms beside power saving hardware components.
Simple uncoordinated seeding of nodes yields a stochastic distribution of nodes after deployment phase. This impedes the assignment of a measured value to its location. Due to this fact, a position determination of all nodes is necessary which, however, consumes additional energy for calculations and data transmissions.
Positioning algorithms are classified into coarse and fine-grained localization, respectively. Fine-grained localization facilitates high precision of position determination but results in expensive calculations and partly high network traffic [3]. However, exact positions are not always required. Often, a

deviation of 7% is sufficient which can be achieved by using coarse-grained algorithms. They merge low calculation requirements with less network traffic.
The optimization of these algorithms is the main focus of our paper. We present a method to optimize important parameters such as the transmission range of base stations (further on defined as beacons) by means of positioning error prior distribution of nodes without simulation.
This paper is subdivided as follows. In Section 2, we explain the "Coarse Grained Localization" algorithm. Then, we present our approach to use this algorithm optimally concerning power constraints and positioning accuracy. We derive an analytic equation to determine the optimal transmission range of beacons in Section 3. Next in Section 4, we describe simulation results to verify the derived equation. In Section 5, we perform further considerations regarding stochastical distribution of beacons. Finally, the paper ends with a conclusion in Section 6.

2 Coarse Grained Localization

In 1989, an algorithm of Coarse Grained Localization (CGL) was successfully implemented in the "Active Badge System" for indoor use based on infrared technique [4]. Years later, Bulusu proposed the Coarse Grained Localization with Center Determination (CGLCD) [5]. It allows the use of CGL algorithms in outdoor environments. This new algorithm is the basis of our parameter optimization.
A sensor network using CGLCD consists of l simple nodes and b beacons. Beacons are sensor nodes knowing their own position $(x_1 \ldots x_b);(y_1 \ldots y_b)$ within the network. The beacons $B_1 \ldots B_b$ are deployed in a grid-aligned network (infrastructure-case) with homogeneous distance d to each other. As simplification, we consider a two-dimensional, quadratic array with width w. Simple sensor nodes may be randomly spread out over the whole array. They do not know their own positions.
Beacons periodically send broadcast messages containing their position. All nodes within transmission range of beacons receive these messages. The messages are assumed to be sent without interfering each other to avoid collisions and packet loss. Every node observes the transmission medium for a defined time period t registering all received messages. Then, it determines the number of beacons n in range. Nodes which are not able to determine their own positions are called unknowns.

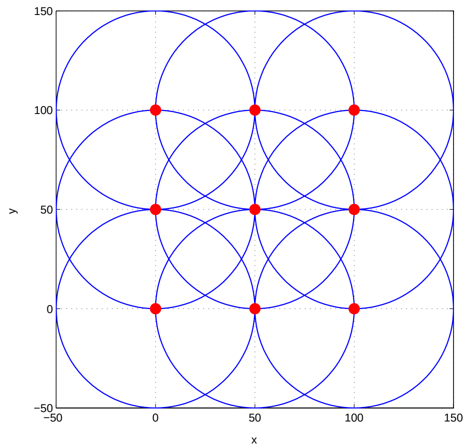

Figure 1: Overlapping regions in a 3x3 array with 9 beacons (solid red circles)

CGLCD uses an idealized radio model to determine the transmission range. Perfect spherical radio waves with radius r and identical transmission ranges for all beacons are assumed and complete the idealized radio model. With the assumption of an idealized circular transmission range of the beacons, it is possible to divide the array into overlapping regions. Depending on the distribution of beacons, regions with *0…b* overlaps are formed (Figure 1). Every node within an overlapping area determines its position approximately based on the received beacon coordinates simply through center determination. The approximated position is given as center of the resulting area of all neighbor beacons as shown in Equation 1.

$$x_{i_{app}}, y_{i_{app}} = \left(\frac{1}{n} \sum_{k=1}^{n} x_{B_k}, \frac{1}{n} \sum_{k=1}^{n} y_{B_k} \right) \quad (1)$$

$x_{i_{app}}, y_{i_{app}}$ = Approximated coordinates of node *i*

The positioning error is defined as distance between the approximated and the exact position of the node as represented in Equation 2.

$$f_i(x, y) = \sqrt{(x_{i_{app}} - x_{i_a})^2 + (y_{i_{app}} - y_{i_a})^2} \quad (2)$$

f_i = Positioning error of node i; x_{i_a}, y_{i_a} = Exact coordinates

Figure 2 shows the distribution of the positioning error over an array of 3x3 beacons with 101x101 nodes. Notice, the error is maximal in the vicinity of beacons due to center determination of resulting rectangles (Equation 1).

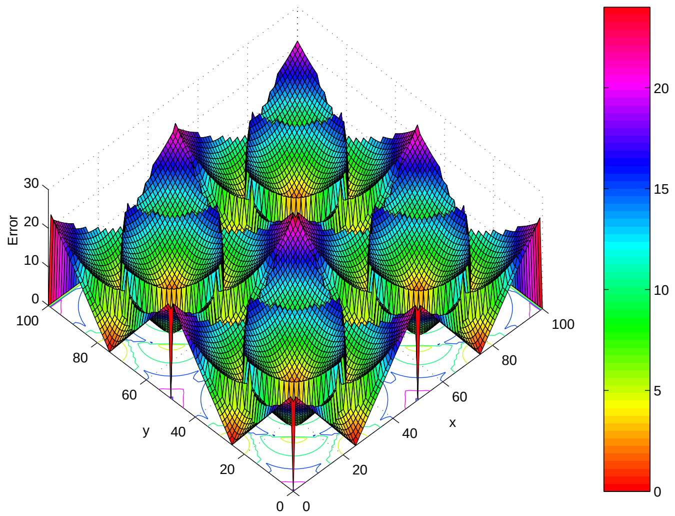

Figure 2: Error distribution over an array of 3x3 beacons, transmission range=50, array width=100. The beacons are arranged as in Figure 1.

3 Low Power Optimization

3.1 Problem Statement

Bulusu gives some real application examples but no further analytical verification. A short analysis can show that, for example, the number of available beacons and their transmission range considerably affect localization error and energy consumption of the network. Therefore, we derive optimal parameters for given constraints, which minimize localization errors and power consumption thus prolonging network availability at no further cost. Beyond, we show that with careful parameter optimization the beacon transmission range can be reduced by 80% thus significantly extending beacon lifetime.

Figure 3 illustrates sensor network scenarios with beacons using different transmission ranges. Figure 3a shows a worst-case scenario with a very low transmission range. It is impossible for most of the nodes to receive messages from beacons. Thus, they are in no overlapping region and can not determine their position – they are unknowns. Complementary, a very high range is useless as demonstrated in Figure 3c because all nodes are within one overlapping region. Consequently, all nodes determine their own position in the center of the whole array. In both cases, the mean error reaches a maximum. It is obvious that there must be exist an optimum of the transmission range as exemplarily shown in Figure 3b. In this scenario, most of the nodes can compute their position based on information received from neighboring beacons.

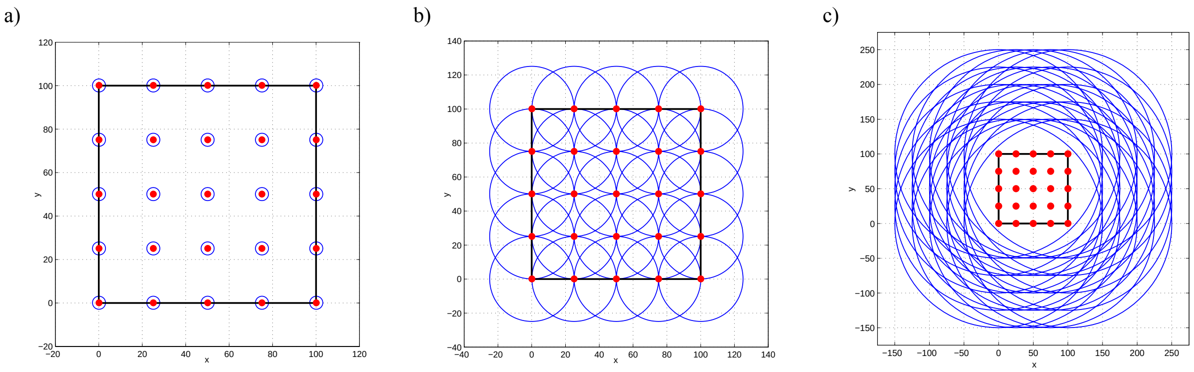

Figure 3: Three scenarios showing a sensor network consisting of randomly distributed simple nodes within the solid rectangles (not shown) and 5x5 beacons with a) low, b) optimal, c) high transmission ranges. In all cases, simple nodes try to determine their own position based on positioning information received from beacons.

It is important to find the optimum transmission range not only with respect to the positioning error but also to reduce power consumption. Based on well known field equations, we can calculate the sending power effort as shown in Equation 3. The formula consists of a static term E_{Init} and a dynamic term E_{Dyn}. We concentrate on E_{Dyn} because E_{Init} depends on technology issues and can not be affected by CGL optimizations.

Equation 3 shows that, in theory, the required energy for sending a message quadratically increases with the transmission range r. In reality, it even raises with the power of 4 due to obstacles and interferences. For our analysis, all parameters in E_{Dyn} except the transmission range r remain constant. Due to the fact that the range is the dominant part in the equation of E_{Dyn}, all parameters and constants are substituted with 1 to make further considerations possible.

$$
\begin{aligned}
E_{Send} &= E_{Init} + mE_{Dyn} \\
E_{Dyn} &= E_{Bit} \cdot \left(\frac{4 \cdot \pi \cdot r}{\lambda} \right)^2 \\
E_{Dyn}\Big|_{E_{Bit} \cdot \left(\frac{4 \cdot \pi}{\lambda} \right)^2 = 1} &= r^2
\end{aligned}
\tag{3}
$$

m = Number of bits to transmit
E_{Init} = Energy to initialize transmitter
E_{Dyn} = Energy required to transmit one bit depending on r
λ = Wave length
r = Transmission range

Both, the positioning error and the energy consumption, depend on the transmission range. Therefore, a benchmark is required to confirm optimizations. We introduce a new metric – the Power-Error-Product (PEP) as derived in Equation 4.

$$\begin{aligned} PEP &= E_{Dyn} \cdot f_{l_{mean}} \\ &= r^2 \cdot \frac{\sum_{i=1}^{l} f_i}{l} \end{aligned} \quad (4)$$

PEP = Power-Error-Product
l = Number of nodes in network
r = Transmission Range
$f_{l_{mean}}$ = Averaged positioning error in sensor network

Figure 4 shows the PEP for *9x9* beacons with several local minima. It is obvious that only the minimum transmission range at *r=11* is reasonable. For less transmission ranges, the algorithm does not operate reliable and too few nodes can determine their position due to missing positioning information as shown in Figure 3a. For higher transmission ranges, the energy consumption increases with the power of 2 as shown in Equation 4. As a consequence, the minimum close to the transmission range with nearly null unknowns must be determined.

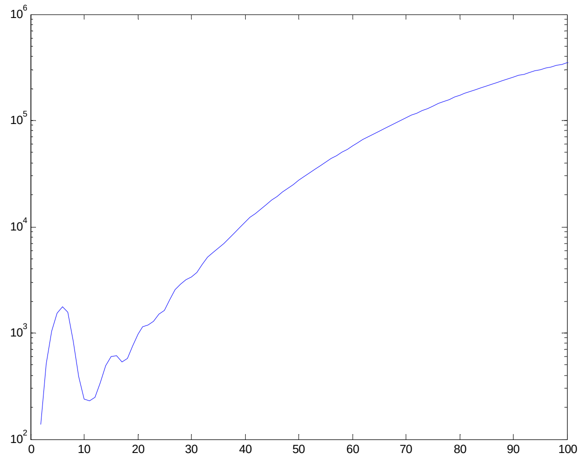

Figure 4: Power-Error-Product depending on transmission range, 9x9 beacons, array width=100

3.2 Analytical Analysis

In case of infrastructure, beacons form a grid array with equal distances to each other as illustrated in Figure 1. Using a given array of width *w* and *n* beacons, the constant distance *d* between the beacons can be determined with Equation 5. For a correct use, we assumed the array to be quadratic and the square root of *n* beacons must be an integer.

$$d = \frac{w}{\sqrt{n} - 1} \qquad n \in N, 4 \leq n < \infty \quad (5)$$

The relationship between distance *d* and transmission range *r* form the granularity *G* as demonstrated in Equation 6.

$$G = \frac{r}{d} \tag{6}$$

Every node determines its position with an approximation error due to the characteristics of the algorithm. The error is zero if the coordinates of the node are equal to the averaged coordinates of all used beacons. The error varies with the geometric characteristics of the field. On the one hand, varying the number of beacons can control the error. On the other hand, the error depends on the transmission range of the beacons, because different numbers of beacons are used for positioning. As already mentioned in Section 3.1, in case of a theoretical infinite transmission range of beacons all nodes determine the same position. Thus, the averaged error is maximal. Otherwise, if the transmission range is zero, none of the node receives a beacon message and the positioning error is maximal, too. Therefore, it must be possible to find a transmission range, which minimizes the positioning error over the entire network. This range is defined as optimal transmission range r_{opt}. The optimal range can be determined under the condition of the existence of an optimal granularity G_{opt}. Equation 7 is formed to calculate the optimal transmission range by insertion of Equation 5 in 6.

$$r_{opt}(w,n) = \frac{G_{opt} \cdot w}{\sqrt{n} - 1} \tag{7}$$

r_{opt} = Optimal transmission range
G_{opt} = Optimal granularity

In contrast to considerations in Section 3.1 which deals with varying the transmission range, now the limiting value of the transmission range r_{opt} is considered in case of, theoretically, unlimited number of beacons *n*. As one can see in Equation 8, the transmission range converges to 0 with $n \rightarrow \infty$.

$$\lim_{n \to \infty} r_{opt}(w,n) \Big|_{w \cdot G_{opt} = const} = \frac{G_{opt} \cdot w}{\sqrt{n} - 1} = \frac{1}{\infty} = 0 \tag{8}$$

4 Simulation

We verified the theoretical assertions in simulations with Matlab 6.5. The dimensions of the grid array are *100x100* in all simulations. We calculated the positioning error for 10201 (*101x101*) simple nodes and determined the average positioning error of all nodes in the sensor network.

In the first simulation, the correlation of transmission range *r*, the number of beacons, and the positioning error is examined basing on the original algorithm without using analytic equations (Figure 5a). Obviously, starting with a high value, the positioning error decreases very fast to a minimum by increasing the transmission range. Then, the error increases nearly linear with the transmission range. However, the objective is a high precision with few beacons and, if possible, a small transmission range. That leads to the determination of the optimal range. As shown in Figure 5, the optimal transmission range decreases with an increasing number of beacons and reduces the positioning error at *10x10* beacons to 1.89%. These considerations result in an optimization problem between number of beacons, positioning error, and transmission range.

Figure 5a illustrates the error behavior caused by an increasing transmission range. Generally, oversizing the transmission range is unfavorable and leads to an increased error and, additionally, energy waste. Using only four beacons, the noticeable shape of the shown curve proves an unfavorable relationship between transmission range, beacons, and size of array. Thus, too few beacons are not recommended. As one can see, the positioning error varies and heavily depends on the transmission range. With careful optimization of parameters, the error can be reduced from 35 to 5 equal to savings of 80%.

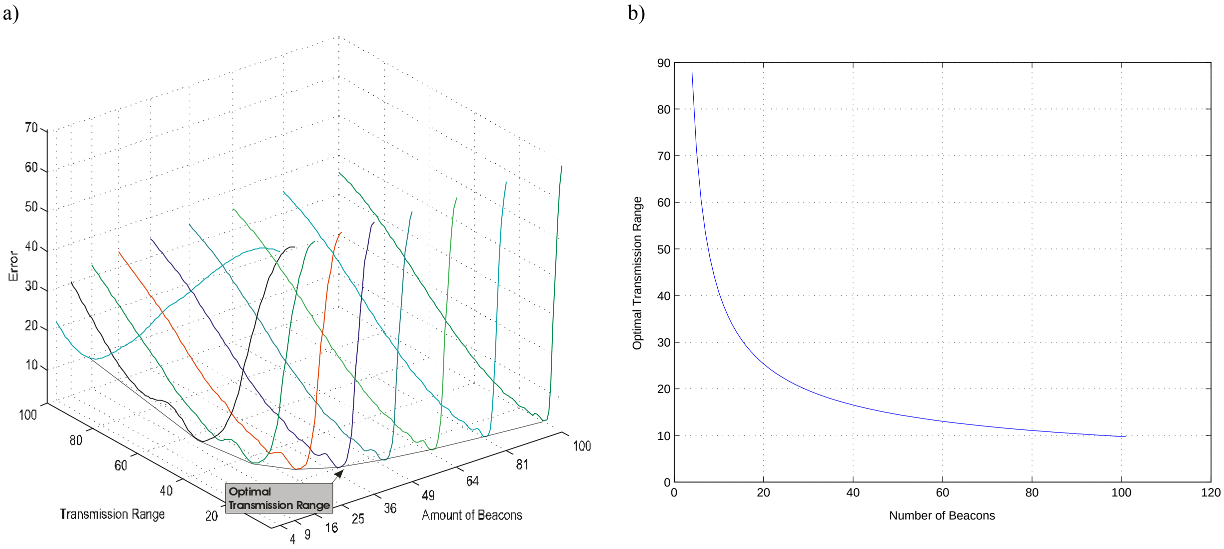

Figure 5: a) Dependency of positioning error on number of beacons and transmission range, b) Optimal transmission range over number of beacons, 101x101 nodes at array width=100

In every simulation, the lowest positioning error is achieved at a granularity of 0.88. Figure 5b demonstrates the optimal transmission range over number of beacons based on Equation 7 using the experimentally found granularity $G_{opt}=0.88$.

With the developed equations from Section 3.2, the Power-Error-Product can be determined over an increasing transmission range (Figure 6a). As described in Section 3.1, the first local minimum of the transmission range at $r<2$ is unusable due to producing too much unknowns during positioning. The determination of the smallest usable transmission range can be done using the graph shown in Figure 6b. This simulation shows the number of unknowns depending on the transmission range. The crossing of the graph with the x-axis shows the smallest possible transmission range $r_{small}=18$ concerning energy consumption. But Equation 7 calculates the optimal transmission range at $r_{opt}=22$ which increases energy consumption but minimizes the positioning error and is the most efficient trade-off.

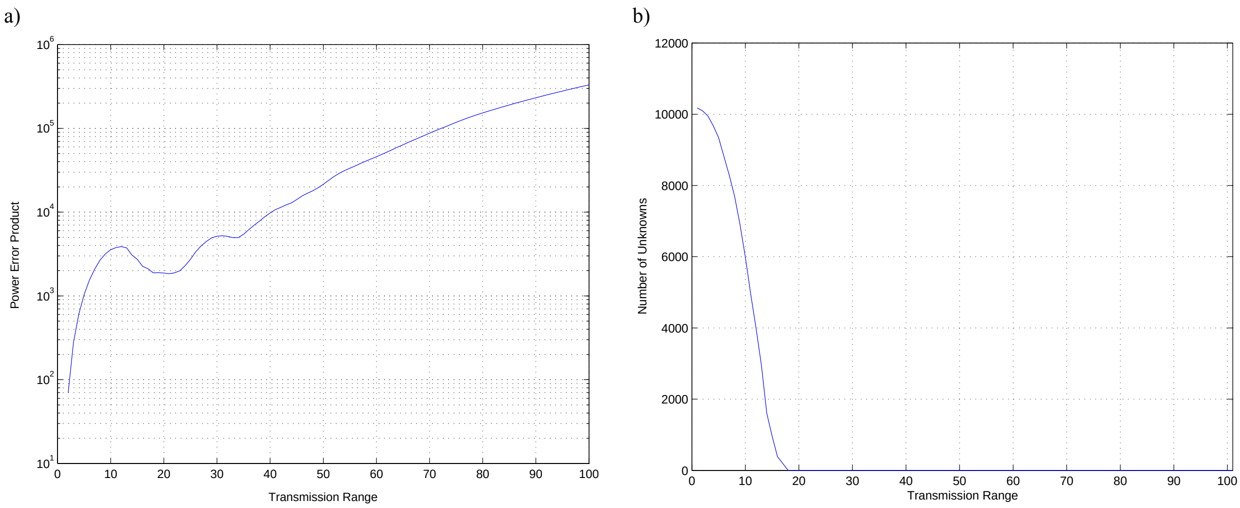

Figure 6: a) Power-Error-Product depending on transmission range, b) Number of unknowns, 5x5 beacons and array width=100

In our simulation, r_{opt} is verified and, of course, is a minimum in the PEP graph shown in Figure 6a. That means the optimal transmission range can be determined using Equation 7 or graphically by choosing the minimum in the Power-Error-Product at a transmission range greater or equal than r_{small}.

5 Stochastical Distribution of Beacons

This section considers the applicability of the parameter optimization in case of stochastically distributed beacons. To compare the simulations of stochastically distributed environments with the results of the infrastructure case, the positioning error over transmission range are determined similarly. In the simulation, we use three different test series of 36 uniformly distributed beacons at a constant number of beacons as shown in Figure 7a. Although the beacons have different positions in every distribution, all graphs are nearly identical and similar to the graphs in the infrastructure case (Figure 5a). Nevertheless, the uniform distribution of the beacons leads to an increased optimal transmission range as well as a loss of precision. With 36 beacons, the positioning error reaches its minimum between 7.9% and 9.4% (Figure 7a) compared to 3.3% (Figure 5a) in the infrastructure case. Thus, a stochastical distribution effects a triplication of the positioning error.

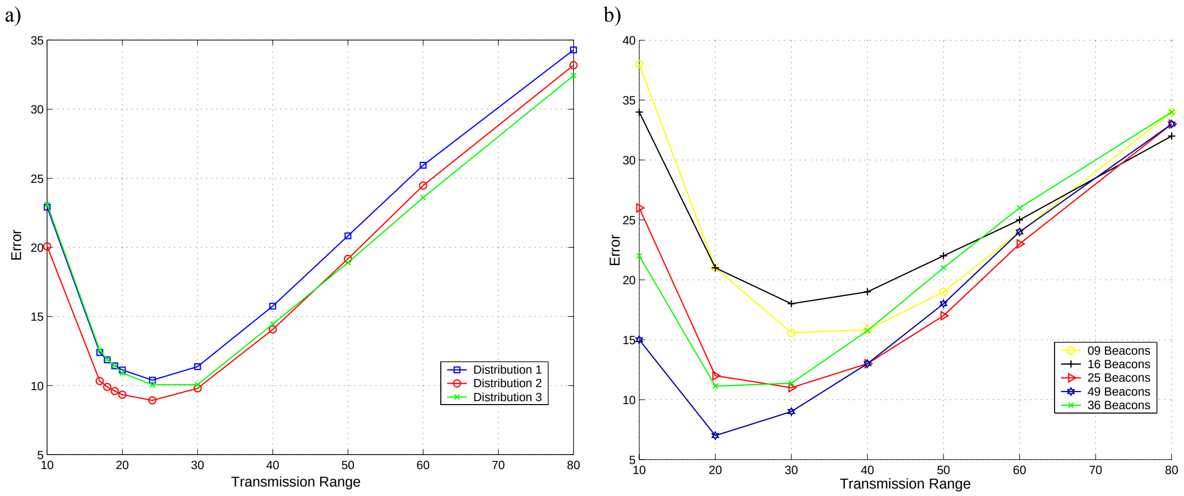

Figure 7: a) Dependency of positioning error on transmission range with different stochastical distributions using 36 beacons, b) Dependency of positioning error on transmission range with different number of beacons

Furthermore, we simulated the positioning error over different number of beacons (Figure 7b). The error and the optimal transmission range decrease with increasing number of beacons. This behavior is similar to the results known from the infrastructure case.

6 Conclusion

In this paper, we presented an analytic solution for calculating an optimal transmission range of beacons for the Coarse Grained Localization algorithm in wireless sensor networks. This knowledge enables reduction of energy consumption and calculation overhead during position determination of simple nodes. Further on, the relative positioning error across the entire sensor network can be significantly reduced.

The presented equations are verified with simulations in Matlab and guarantee the determination of an optimal transmission range only for grid-aligned beacons. Therefore, our current activities concentrate on analytic solutions for a stochastical distribution of beacons.

7 References

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