Error Analysis of Single Frequency GPS Measurements and Impact on Timing and Positioning Accuracy

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Abstract – The Instantaneous Pseudo Range Error (IPRE) for a single frequency GPS receiver is defined as the contribution of major individual error sources (satellite clock, ephemeris, ionosphere, troposphere, multipath and receiver noise) to the pseudo range fluctuations for a given epoch. From the principle that considers an error as a deviation of an estimate from a reference value, it is possible to determine individual errors as function of time. For this, a single frequency receiver at different IGS locations is simulated using post-processed IGS data as reference. The behavior of errors is studied and their impact on timing and positioning accuracy is analyzed.

1 Introduction

This paper analyzes the behavior of individual errors grouped in 5 categories (satellite clock error, ephemeris error, ionospheric error, tropospheric error and multipath + receiver noise error)
As a first step, we will define the error model retained, the error principle and the methodology of error calculation. In a second step we will present the results of a statistical analysis at pseudo range level. In a third step we will analyze the impact of these errors on timing and positioning accuracy.

2 Error formulation

Let us recall the formulation of the Instantaneous Pseudo Range Error (IPRE) as function of instantaneous individual errors defined in [1]:

$$\overline{\mathbf{IPRE}} \equiv -\overline{\mathbf{Clk}} + \overline{\mathbf{Eph}} + \overline{\mathbf{Iono}} + \overline{\mathbf{Trop}} - \overline{\mathbf{MN}} = \overline{\overline{\mathbf{G}}} \cdot \overline{\Delta \mathbf{x}}, \qquad (1)$$

where
$(\bar{\ })$ represents a vector for which each element represents an observation
$(\bar{\bar{\ }})$ represents a matrix notation
$\overline{\mathbf{IPRE}}$ is the vector of instantaneous pseudo range error.
$\overline{\mathbf{Clk}}$ is the vector of satellite clock error.
$\overline{\mathbf{Eph}}$ is the vector of ephemeris error.
$\overline{\mathbf{Iono}}$ is the vector of ionospheric error.
$\overline{\mathbf{Trop}}$ is the vector of tropospheric error.
$\overline{\mathbf{MN}}$ is the vector of combined multipath and receiver noise error.

$$\overline{\overline{\mathbf{G}}}_{K\times 4} = \begin{bmatrix} \hat{\overline{\mathbf{1}}}_{s1}^{T} & 1 \\ \hat{\overline{\mathbf{1}}}_{s2}^{T} & 1 \\ \vdots & \vdots \\ \hat{\overline{\mathbf{1}}}_{sK}^{T} & 1 \end{bmatrix}, \quad \overline{\Delta\mathbf{x}}_{4\times 1} = \begin{bmatrix} \overline{\Delta\mathbf{r}_{\mathbf{u}}} \\ -c\cdot\Delta b_u \end{bmatrix}$$

with $\overline{\mathbf{1}}_s$ corresponding to the unit vector from user to satellite, K represents the number of satellites on visibility (at least 4 are required to solve the position and time equation).
$\overline{\Delta\mathbf{r}_{\mathbf{u}}}$ represents the vector of user position error, c the speed of light and Δb_u the error on the user clock bias.

The error is defined as a deviation of an estimated value from a reference value: error (t) = estimate (t) – reference (t). The following table lists the data chosen for estimation of individual error components and as reference.

Error type	**Estimate**	**Reference**
Clk	Navigation message	SP3 files
Eph	Navigation message	SP3 files
Iono	Navigation message + Klobuchar model	IONEX files
Trop	MOPS model + Neill's mapping function	SINEX files
MN	No estimation	Results of the use of TEQC program

Tab. 1: Models and data sources for estimates and reference values.

In the following we summarize the analysis of one year measurements (2003) using data of 7 IGS stations with locations shown in Fig. 1.

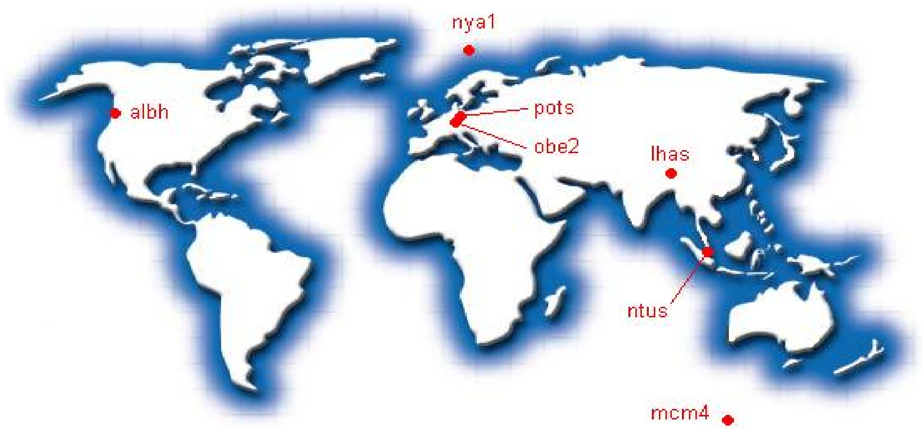

Fig. 1: Geographic locations of IGS stations.

3 Analysis of results at pseudo range level

One of the first results we can obtain is the UERE (User Equivalent Range Error) determined at each IGS station as root mean square (rms) of IPRE taken at different times *t*:

$$UERE = \sqrt{\frac{1}{N}\sum_{t=1}^{N} IPRE_t^2}$$

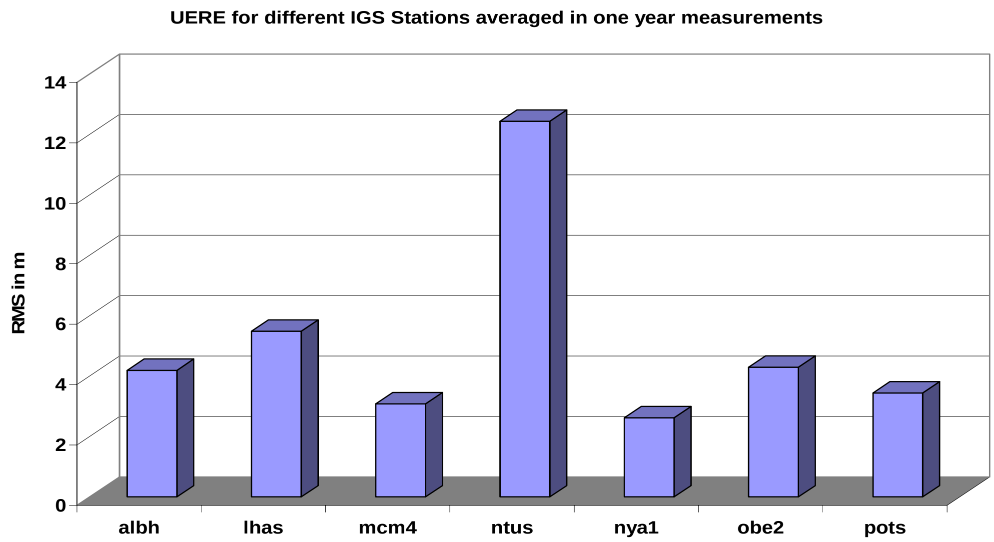

Fig. 2: UERE for IGS stations.

The first observation we can notice is the high sensibility of the UERE to the geographic location of the user. Ntus (located at Singapore) near the equator is highly influenced by the high ionospheric activity in this region and the Klobuchar correction model can hardly correct this effect.
A second result is obtained by comparing the variance of total IPRE from our measurements with the sum of variances of individual errors (as if individual errors were uncorrelated). Fig.3 is obtained by calculating $[\mathrm{var}(Clk)+\mathrm{var}(Eph)+\mathrm{var}(Iono)+\mathrm{var}(Trop)+\mathrm{var}(MN)]-\mathrm{var}(IPRE)$ in percent of $\mathrm{var}(IPRE)$. The usual assumption that considers individual errors as uncorrelated (see [2]) finds here its limits.

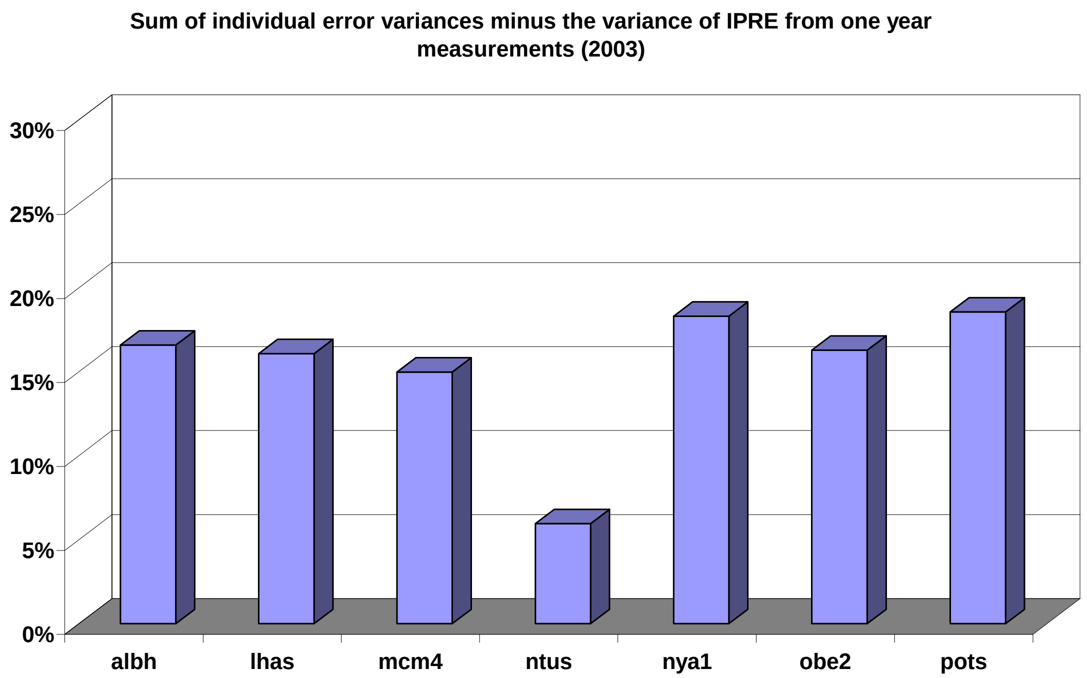

Fig. 3: Sum of individual error variances minus the variance of IPRE in %

Fig. 4 represents the distribution of individual errors in percentage of the global error.
The bias operator is linear which means that any bias has an influence in the global IPRE. A bias in any of the individual errors will impact the IPRE at the same magnitude. For better representation we use the absolute value of bias defined as the mean value of one year measurements.

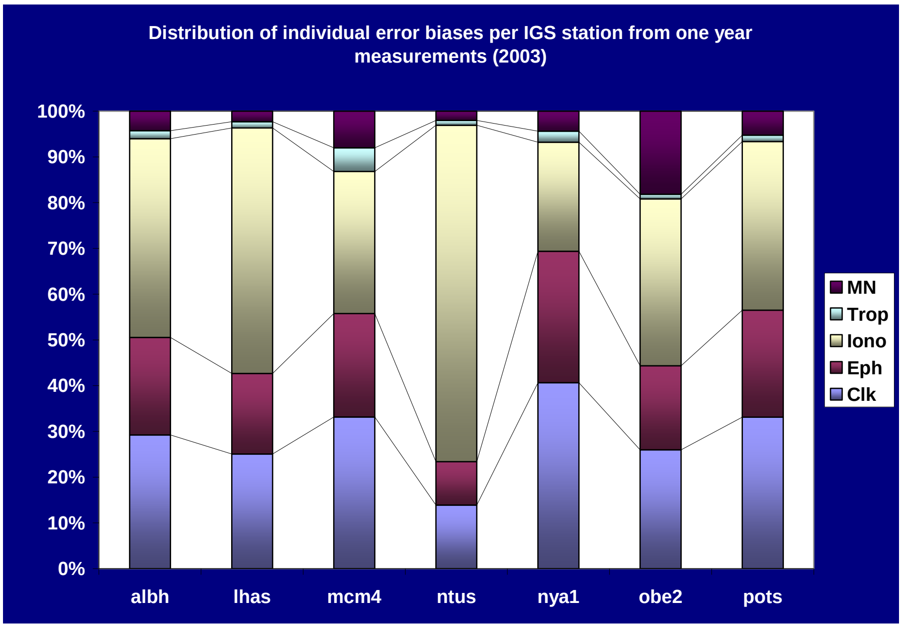

Fig. 4: Distribution of error biases.

A similar procedure is done for the variances of errors, but with the difference that the variance operator is quadratic. This means that the dominant individual error variance will drive the global error. In other words, we can neglect the other effects: The third highest variance represents less than 10 % of the total IPRE variance except for MN at obe2.

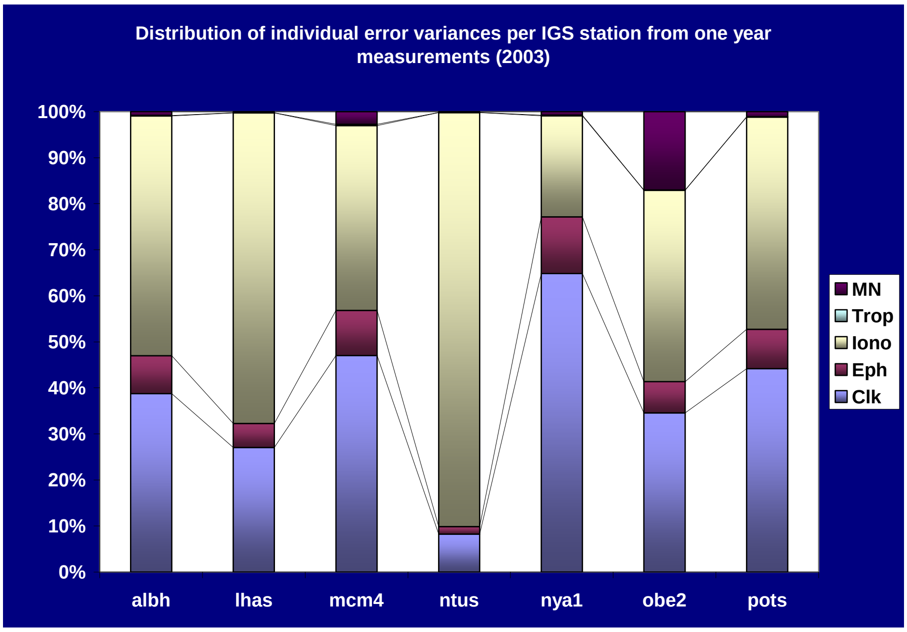

Fig. 5: Distribution of error variances.

4 Impact of individual errors on timing and positioning accuracy

From (1) we can determine the position and time vector $\overline{\mathbf{\Delta x}}$ by multiplying left and right hand side by $\left(\overline{\overline{\mathbf{G}}}^{T}\overline{\overline{\mathbf{G}}}\right)^{-1}\overline{\overline{\mathbf{G}}}^{T}=\overline{\overline{\mathbf{H}}}$. By linearity we obtain:

$$\overline{\mathbf{\Delta x}}=\overline{\overline{\mathbf{H}}}\cdot\overline{\mathbf{IPRE}}=-\overline{\overline{\mathbf{H}}}\cdot\overline{\mathbf{Clk}}+\overline{\overline{\mathbf{H}}}\cdot\overline{\mathbf{Eph}}+\overline{\overline{\mathbf{H}}}\cdot\overline{\mathbf{Iono}}+\overline{\overline{\mathbf{H}}}\cdot\overline{\mathbf{Trop}}-\overline{\overline{\mathbf{H}}}\cdot\overline{\mathbf{NM}} \quad (2)$$

In this equation, we use all satellites on visibility.
The first 3 coordinates of $\overline{\mathbf{\Delta x}}$ correspond to the error of the user position. It is easy to see the impact of each individual error. In the figure below we represent the 2σ error envelops in projection into the 3 fundamental planes (horizontal plane, and 2 vertical planes along West-East and South-North directions) using the local coordinates system.

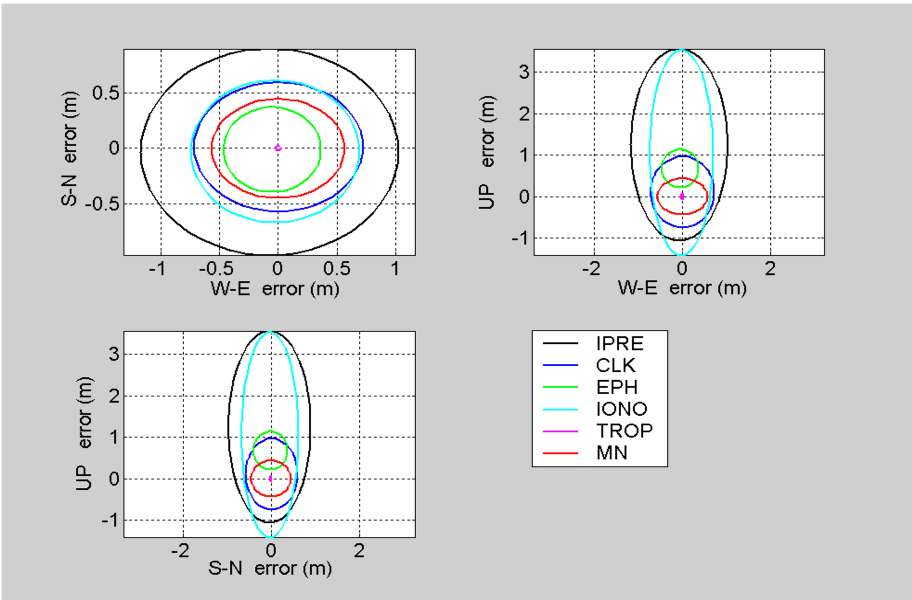

Fig. 6: Repartition of 2σ error envelops at obe2.

Here we can see how the dominant error plays a role in horizontal and vertical position error. In black we can see the IPRE contribution to position error which is simply the complete position error.
The horizontal position error is mostly influenced by the Clk and the Iono error. The horizontal error envelops are almost circular which means that no direction is privileged. In contrast, the vertical error is mainly influenced by the ionosphere error envelop and is 2 to 3 times bigger than the horizontal component (note different scales for horizontal and vertical errors).
The 4th coordinate of $\overline{\mathbf{\Delta x}}$ represents the receiver clock bias error. In the same way as for the position, we analyse the impact of each individual error in the receiver clock error expressed in seconds. In Fig. 7 we represent the statistical distributions of each contribution to the receiver clock error at the IGS station obe2.

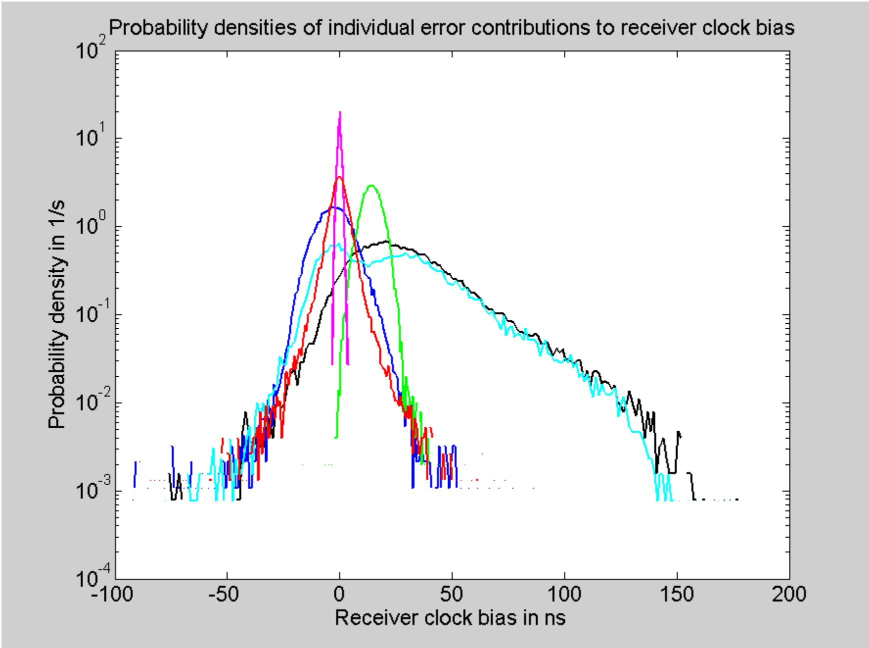

Fig. 7: Distribution of error contributions to receiver clock bias at obe2.

The colour code is the same as in Fig. 6. Here again, as for the vertical position error, the black curve seems to follow the cyan one. The ionospheric error drives the receiver clock bias error.

5 Conclusion

The representation of error as a deviation from a reference and the use of the IPRE concept allow us to make a detailed error analysis which gives promising results. The first one is a regional characteristic of UERE (mainly due to the ionospheric activity). The second is the non negligible part of correlations which overestimate the total pseudo range error (5 to 20%). 2 other interesting results are the bias effect and the variance effect: a bias even small will influence the total pseudo range error, the variance is driven by the 1st or the two 1st dominant error variances.
When looking at the impact on the timing and positioning accuracy, the following conclusion can be made: both vertical error and receiver clock bias are driven by the ionospheric error. Moreover, the vertical error is 2 or 3 times bigger than the horizontal one, also due to the ionospheric error. Therefore, efforts should be focussed on correcting this dominant error in single frequency stand alone receivers.

6 Acknowledgments

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7 References

[1] B. Belabbas, F. Petitprez, A. Hornbostel, UERE Analysis for Static Single Frequency Positioning Using Data of IGS Stations, proceedings of the ION National Technical Meeting, San Diego USA, January 2005

[2] B.W. Parkinson, J. J. Spilker Jr., Global Positioning System: Theory and Applications, Vol. 1, Progress in Astronautics and Aeronautics, 1996.