Rice Factor Estimation for GNSS Reception Sensitivity Improvement in Multipath Fading Environments

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Abstract - Satellite navigation is increasingly considered for use in environments with an attenuated direct path (and delayed echoes). Examples include areas inside buildings as well as street canyons. The attenuation of the signal experienced in such environments poses a problem for acquisition. In relevant environments, the power distribution of the received signal can be modeled by a Rice distribution. In the present paper, we show, how the knowledge of the Rice factor can be used to recover a large portion of the sensitivity loss during the acquisition on fading channels. In the case of a GPS L1-C/A signal, a Rice factor of 1 and a noncoherent integration interval of 1 s. 2.4 dB can be gained with respect to the conventional approach.

1 Introduction

In most situations, the propagation from a satellite to a navigation receiver is subject to reflections, diffraction, and scattering from obstacles. These obstacles are typically near the receiver. Measurement campaigns have rarely shown delays in excess of 500 ns, see [1], [2]. This corresponds to an excess distance up to 150 m. Receivers for the consumer market use the GPS L1-C/A or the Galileo E2-L1-E1 open service signals. In the acquisition mode, such a receiver searches for a correlation result larger than a threshold (Neyman-Pearson criterion). The correlation is typically performed in half chip steps. Correspondingly, in this mode, such a receiver experiences a channel that is well modeled by a Rice distributed attenuation.

In applications, such as ship and airplane guidance (open areas) or car navigation (sensor fusion), the impact of the fading has marginal consequences. In pedestrian urban applications, however, the receiver has to acquire the signal instantaneously in a potentially unfavorable position. The direct path might be severely attenuated. This has led to the development of assistance procedures [5], massively parallel correlation or matched filtering [3], and the additional use of other systems, like cellular [4]. The present technique is another contribution to further improve the situation.

2 Sensitivity Improvement by Estimation of the Multipath Fading Statistics

The threshold in the Neyman-Pearson criterion in signal detection is defined by a false alarm rate that is considered optimum or at least suitable in a given receiver realization. When nu-

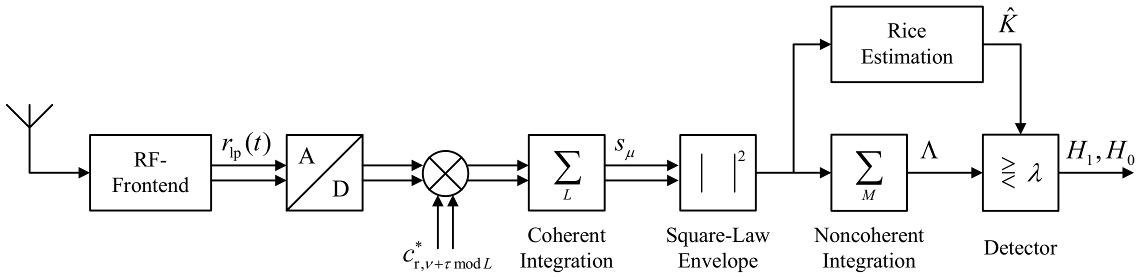

Figure 1: Galileo/GPS receiver channel including Rice estimator.

merical values are needed, we shall consider a setting a probability of false alarm $P_\mathrm{f} = 10^{-3}$. False alarms are caused by out-of-phase autocorrelation, crosscorrelation, and additive noise. Multipath fading reduces both the correlation peak and the out-of-phase autocorrelations. The former effect decreases the detection probability, the latter one decreases the false alarm rate. As a consequence of the decreased false alarm rate, the threshold is no more optimal. Correspondingly, we shall adapt the threshold to the fading and thereby essentially recover the acquisition sensitivity of the receiver. This requires an estimation of the Rice factor and a subsequent adaptation of the threshold. The setting of this threshold and the resulting performance evaluation are described in Section 4. In the next section, we shall describe the statistics of the coherent predection, and in Section 5, we shall comment on the estimation of the Rice factor.

3 Coherent Predetection

The received GPS or Galileo signal can be expressed in its complex-valued, low-pass equivalent form as

$$r_\mathrm{lp}(t) = \sqrt{2C}d(t)c(t)\mathrm{e}^{\mathrm{j}(2\pi\Delta ft+\phi)}v(t) + n(t), \tag{1}$$

where C denotes the received signal power, $d(t)$ the data modulation, $c(t)$ the received spreading code, Δf the frequency deviation, ϕ the phase offset, $v(t)$ the complex fading attenuation, and $n(t)$ complex-valued, zero-mean, white Gaussian noise with variance

$$\sigma_n^2 = \mathrm{E}\left\{|n|^2\right\} = 2\mathrm{E}\left\{\Re\{n\}^2\right\} = 2\mathrm{E}\left\{\Im\{n\}^2\right\} = 2N_0BF. \tag{2}$$

N_0 denotes the thermal noise power spectral density, F the receiver noise figure, and $B = 1/T_\mathrm{s}$ the bandwidth of the signal for the sample period T_s.

The fading attenuation $v(t)$ in (1) is a nonzero-mean, complex-valued Gaussian process. The envelope $v(t)$ is Ricean distributed

$$p_{|v|}\left(|v|\right) = \frac{2|v|}{\sigma_v^2}\exp\left(-\frac{|v|^2 + A_v^2}{\sigma_v^2}\right)\mathrm{I}_0\left(\frac{2A_v|v|}{\sigma_v^2}\right), \tag{3}$$

$$A_v = \left|\mathrm{E}\{v\}\right|, \tag{4}$$

$$\sigma_v^2 = \mathrm{E}\left\{|v - A_v|^2\right\} = 2\mathrm{E}\left\{\Re\{v - A_v\}^2\right\} = 2\mathrm{E}\left\{\Im\{v - A_v\}^2\right\}, \tag{5}$$

with $\mathrm{I}_0(x)$ being the modified Bessel function of first kind and zero order. The ratio between the deterministic signal power of the line-of-sight (LOS) component and the variance of the multipath component is the Rice factor

$$K = \frac{A_v^2}{\sigma_v^2}. \tag{6}$$

A weak LOS path corresponds to a low Rice factor and a strong LOS propagation path to a large Rice factor. Also note that since the received signal power C has been factored out in Equation (1), we have

$$A_v^2 + \sigma_v^2 = 1$$

Despreading with the local PRN reference code $c_{\mathrm{r},\nu}$ and coherent integration of $L = T_\mathrm{i}/T_\mathrm{s}$ chips, with T_i being the coherent integration time, yields

$$s_\mu = \sqrt{2C} \sum_{\nu=\mu L}^{(\mu+1)L-1} d_\nu c_\nu c^*_{\mathrm{r},\nu+\tau \bmod L} \mathrm{e}^{\mathrm{j}(2\pi\Delta f \nu T_\mathrm{s}+\phi)} v_\nu + \sum_{\nu=\mu L}^{(\mu+1)L-1} n_\nu c^*_{\mathrm{r},\nu+\tau \bmod L}. \tag{7}$$

For a sufficiently small frequency deviation Δf, the signal phase can be approximated by its average value during each coherent integration interval $[\mu T_\mathrm{i},\ (\mu+1)T_\mathrm{i}]$

$$\frac{1}{L} \sum_{\nu=\mu L}^{(\mu+1)L-1} \mathrm{e}^{\mathrm{j}(2\pi\Delta f \nu T_\mathrm{s}+\phi)} \approx \frac{1}{T_\mathrm{i}} \int_{\mu T_\mathrm{i}}^{(\mu+1)T_\mathrm{i}} \mathrm{e}^{\mathrm{j}(2\pi\Delta f t+\phi)}\ \mathrm{d}t = \mathrm{sinc}\left(\Delta f T_\mathrm{i}\right) \mathrm{e}^{\mathrm{j}(\pi\Delta f(2\mu+1)T_\mathrm{i}+\phi)}. \tag{8}$$

Without unknown data bit transitions within the interval $[\mu T_\mathrm{i},\ (\mu+1)T_\mathrm{i}]$, the coherent predetection results in

$$s_\mu \approx \sqrt{2C} d_\mu R_{rc}(\tau) \mathrm{sinc}\left(\Delta f T_\mathrm{i}\right) \mathrm{e}^{\mathrm{j}(\pi\Delta f(2\mu+1)T_\mathrm{i}+\phi)} v_\mu + w_\mu, \tag{9}$$

where $R_{rc}(\tau)$ is the crosscorrelation function between the received and locally generated spreading codes. The approximation above assumes that the coherent integration time T_i is chosen to be no longer than the coherence time of the multipath propagation channel. This results in a fading that is essentially constant during each of the integration intervals. By making the additional assumption that the fading is uncorrelated between integration intervals, the computations become mathematically tractable. The distribution of v_μ then becomes independent of μ and is fully described by the Rice factor K. From Equation (6), and (3), we have

$$A_v^2 = \left|\mathrm{E}\{v\}\right|^2 = \frac{K}{K+1}, \tag{10}$$

$$\sigma_v^2 = \mathrm{E}\left\{|v - A_v|^2\right\} = 2\mathrm{E}\left\{\Re\{v - A_v\}^2\right\} = 2\mathrm{E}\left\{\Im\{v - A_v\}^2\right\} = \frac{1}{K+1}. \tag{11}$$

The complex-valued, zero-mean, white Gaussian noise w_μ is an accumulation of L statistically independent noise samples and therefore has the variance

$$\sigma_w^2 = \mathrm{E}\left\{|w|^2\right\} = 2\mathrm{E}\left\{\Re\{w\}^2\right\} = 2\mathrm{E}\left\{\Im\{w\}^2\right\} = 2N_0\frac{1}{T_\mathrm{s}}F = 2N_0\frac{L^2}{T_\mathrm{i}}F. \tag{12}$$

4 Detector Test Statistic

The Neyman-Pearson criterion maximizes the probability of detection

$$P_\mathrm{d} = \int_\lambda^\infty p_{\Lambda|H_1}\left(\Lambda|\, H_1\right)\ d\Lambda. \tag{13}$$

for a given probability of false alarm P_f. The threshold λ is therefore calculated for a fixed probability of false alarm

$$P_\mathrm{f} = \int_\lambda^\infty p_{\Lambda|H_0}\left(\Lambda|\, H_0\right)\ d\Lambda \tag{14}$$

Table 1: Sensitivity gain through known Rice factor for $T_\mathrm{i} = 20\,\mathrm{ms}$ and $P_\mathrm{f} = 10^{-3}$.

	$K=5$	$K=2$	$K=1$	$K=0$
$M=50$	1.3 dB	1.9 dB	2.3 dB	2.4 dB
$M=500$	1.6 dB	2.5 dB	3.0 dB	3.3 dB
$M=5000$	1.7 dB	2.8 dB	3.2 dB	3.7 dB

using hypothesis H_0. Out-of-phase autocorrelation $R_{rc}(\tau \neq 0)$ plus noise w are present for H_0, whereas the correlation peak $R_{rc}(0)$ plus noise w is present for hypothesis H_1.

The coherent integration period is limited by the coherence time of the propagation channel and the oscillator accuracy. For enhanced reception sensitivity, the predetection samples s_μ are further integrated noncoherently. The squared envelope is usually applied to minimize the implementation complexity. The resulting detector test statistic

$$\Lambda = \sum_{\mu=0}^{M-1} |s_\mu|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{15}$$

is the sum of M squared Gaussian-distributed variables. With the stated independency of v_μ as a function of μ, the test statistic Λ follows the noncentral Chi-squared distribution

$$p_\Lambda(\Lambda) = \frac{1}{2\alpha^2}\left(\frac{\Lambda}{\gamma^2}\right)^{\frac{M-1}{2}} \exp\left(-\frac{\Lambda+\gamma^2}{2\alpha^2}\right) \mathrm{I}_{M-1}\left(\frac{\sqrt{\Lambda\gamma^2}}{\alpha^2}\right). \tag{16}$$

Each variable s_μ is the sum of two statistically independent, complex-valued Gaussian variables v_μ and w_μ resulting in the combined variance

$$\alpha^2 = CR_{rc}^2(\tau)\mathrm{sinc}^2(\Delta f T_\mathrm{i})\frac{1}{K+1} + \sigma_w^2. \tag{17}$$

$\mathrm{I}_{M-1}(x)$ is the modified Bessel function of first kind and order $M-1$. The Chi-squared distribution has $2M$ degrees of freedom and the noncentrality parameter

$$\gamma^2 = MCR_{rc}^2(\tau)\mathrm{sinc}^2(\Delta f T_\mathrm{i})\frac{K}{K+1}. \tag{18}$$

The Figures 2, 3 and 4 compare the probabilities of detection for the cases where the Rice factor K is known to the cases where K is unknown and Table 1 summarizes the results. When K is known, the detection threshold λ is calculated for H_0 with a Rice factor K equal to the actual Rice factor. When K is unknown, the detection threshold λ is calculated for H_0 with a Rice factor $K \to \infty$ corresponding to a line-of-sight signal. If a lower value of K was assumed, the maximally allowed false alarm rate P_f would be violated for all cases where the actual Rice factor K is higher than the one used to calculate the threshold λ. The maximal out-of-phase autocorrelation values under hypothesis H_0 are set to $R_{rc}(\tau \neq 0) = 65$ for the GPS C/A code, since it is a Gold code with polynomials of degree 10 and a code length of 1023 chips [6]. The correlation peak under hypothesis H_1 is correspondingly $R_{rc}(0) = 1023$.

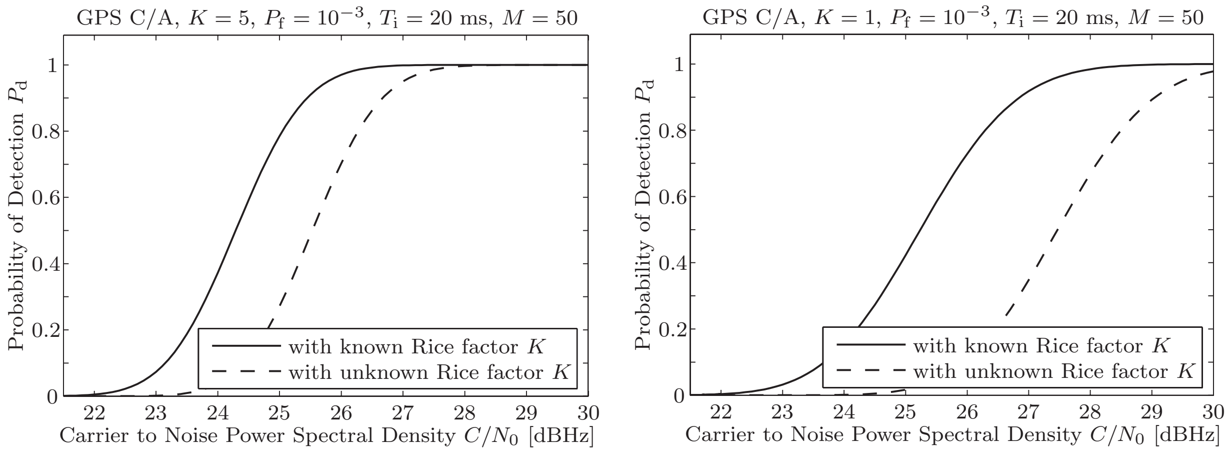

Figure 2: Comparison of the reception sensitivity for known and unknown K with $M = 50$.

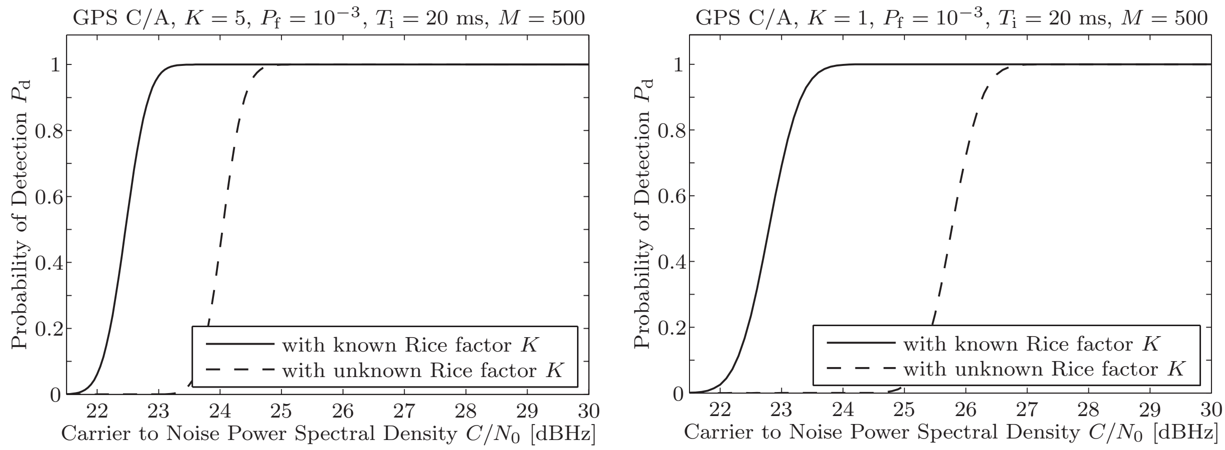

Figure 3: Comparison of the reception sensitivity for known and unknown K with $M = 500$.

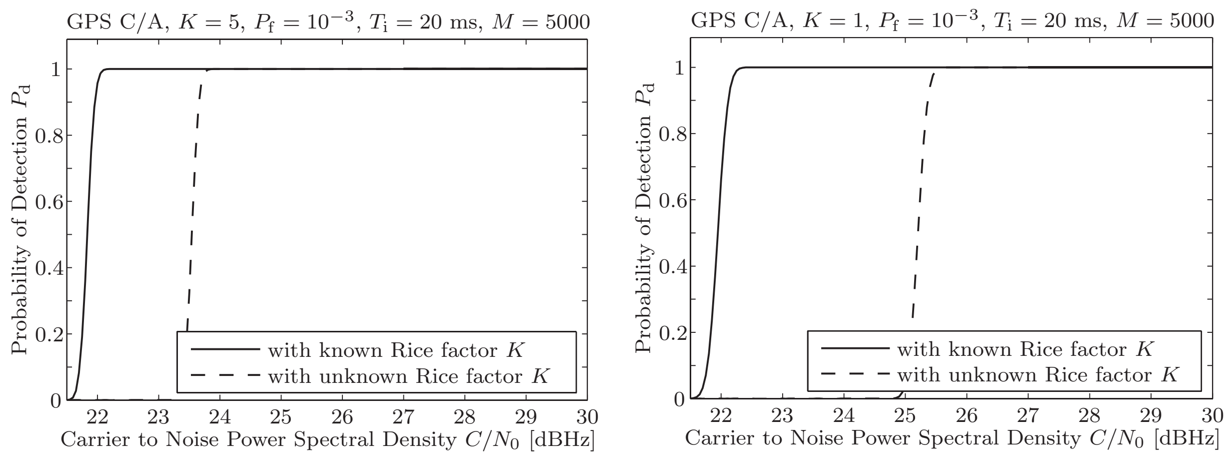

Figure 4: Comparison of the reception sensitivity for known and unknown K with $M = 5000$.

5 Rice Factor Estimation

The Rice factor can be estimated through different methods. The most common techniques are based on statistical moments, maximal likelihood ratio, or least squared error estimations [7]. A method with low hardware implementation complexity, as well as fast and accurate convergence relies on the 2^{nd} and 4^{th} noncentral moments and therefore has the advantage that it only requires samples of the envelope of the received signal and integrates nicely into the receiver processing [8]

$$\hat{K} = \frac{\sum_{\mu=0}^{M-1} |v_\mu|^4 - 2\left(\sum_{\mu=0}^{M-1} |v_\mu|^2\right)^2 - \sum_{\mu=0}^{M-1} |v_\mu|^2 \sqrt{2\left(\sum_{\mu=0}^{M-1} |v_\mu|^2\right)^2 - \sum_{\mu=0}^{M-1} |v_\mu|^4}}{\left(\sum_{\mu=0}^{M-1} |v_\mu|^2\right)^2 - \sum_{\mu=0}^{M-1} |v_\mu|^4}. \tag{19}$$

6 Conclusion

Rice factor estimation provides a sensitivity gain for the acquisition of CDMA signals in multipath fading environments. This gain is easy to realize, and is typically larger than 1.5 dB for the GPS C/A signal in an urban environment.

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